Exercise 19

Let $f(x) = x^2$.

(a) Estimate the values of $f'(0)$, $f'\left(\frac{1}{2}\right)$, $f'(1)$, and $f'(2)$ by using a graphing device to zoom in on the graph of f.

(b) Use symmetry to deduce the values of $f'\left(-\frac{1}{2}\right)$, $f'(-1)$, and $f'(-2)$.

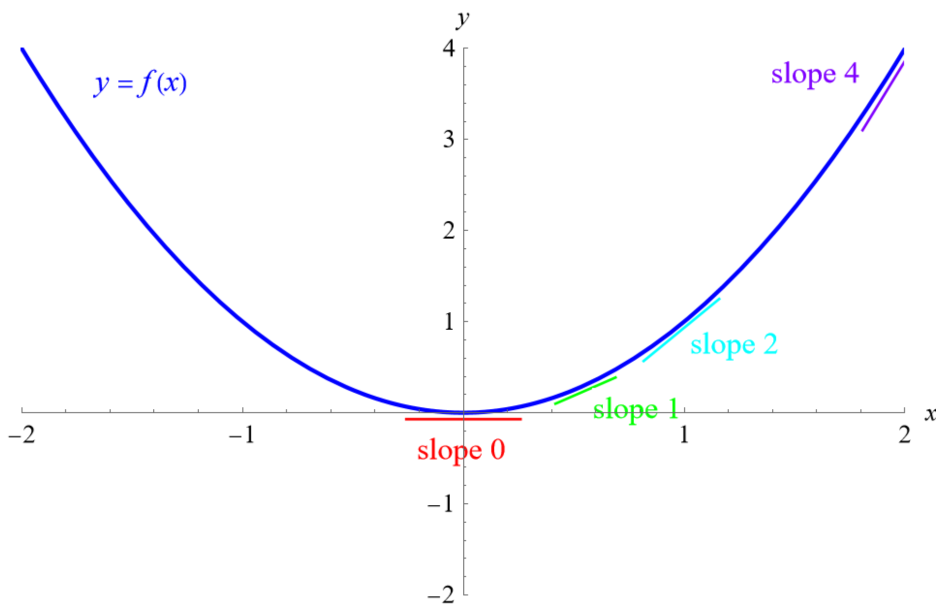
(c) Use the results from parts (a) and (b) to guess a formula for $f'(x)$.

(d) Use the definition of derivative to prove that your guess in part (c) is correct.

Solution

Part (a)

Draw the tangent lines to the graph at the given values of x and estimate their slopes.



As a result,

$$f'(0) = 0$$

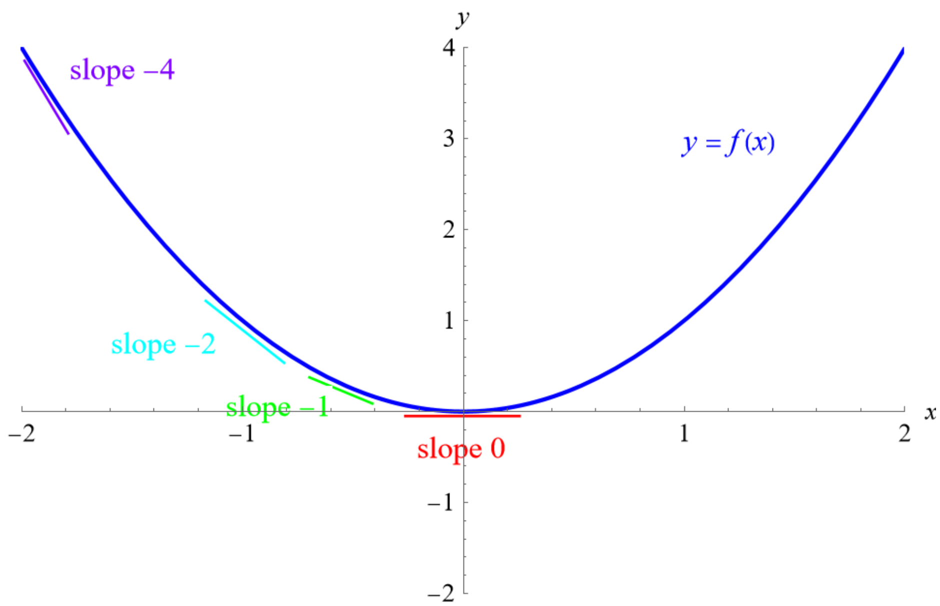
$$f'\left(\frac{1}{2}\right) = 1$$

$$f'(1) = 2$$

$$f'(2) = 4.$$

Part (b)

Draw the tangent lines to the graph at the given values of x and estimate their slopes.



As a result,

$$f'\left(-\frac{1}{2}\right) = -1$$

$$f'(-1) = -2$$

$$f'(-2) = -4.$$

Part (c)

Notice that the slope is double the value of the x-coordinate, so

$$f'(x) = 2x.$$

Part (d)

Calculate the derivative of $f(x) = x^2$ from the definition.

$$\begin{aligned} f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{(x+h)^2 - x^2}{h} &= \lim_{h\to 0} \frac{(x^2 + 2xh + h^2) - x^2}{h} \\ &= \lim_{h\to 0} \frac{2xh + h^2}{h} \\ &= \lim_{h\to 0} (2x + h) \\ &= 2x \end{aligned}$$